Examples 1 in Basic Geometry

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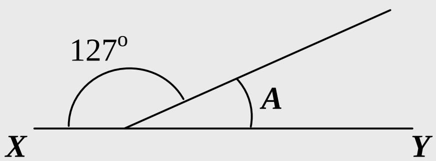
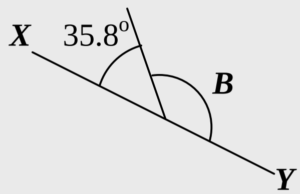
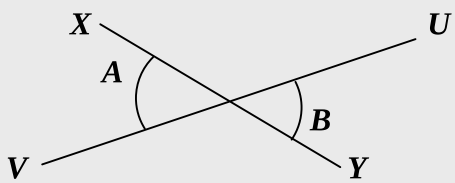
Examples 1

0. Assuming ***XY*** is a line segment, find the two angles ***A*** and ***B***.

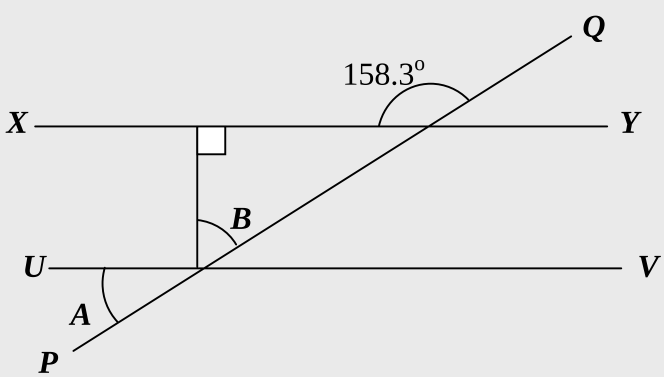




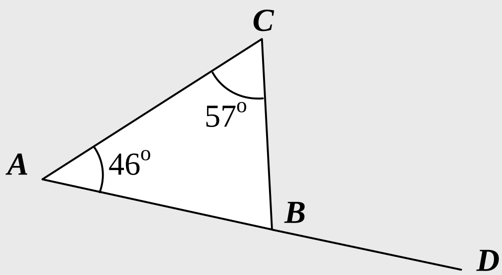
1. Assuming ***XY*** and ***UV*** are two line segments, show that the two angles ***A*** and ***B*** are the same.



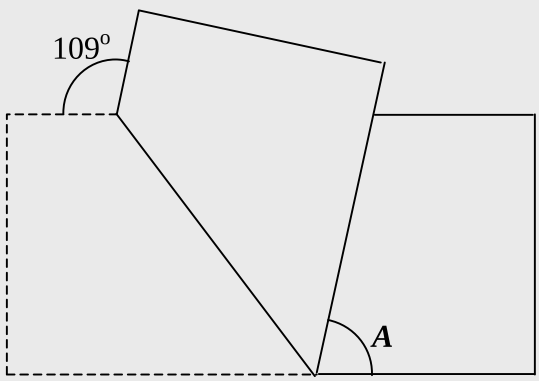
2. Assuming ***XY*** is parallel to ***UV***, find the two angles ***A*** and ***B***.



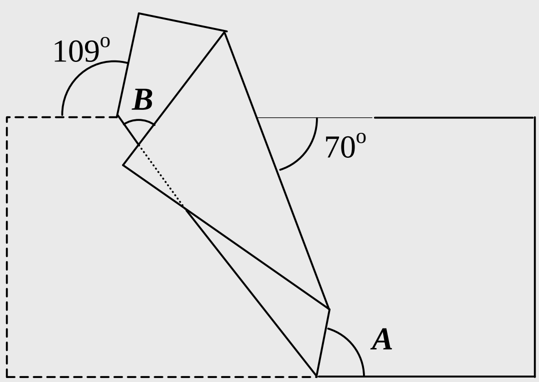
3. Assuming the line segment AD includes the side AB in ΔABC, find $\angle CBD$.



4. Assuming a rectangle gets folded the way below, find the angle A.

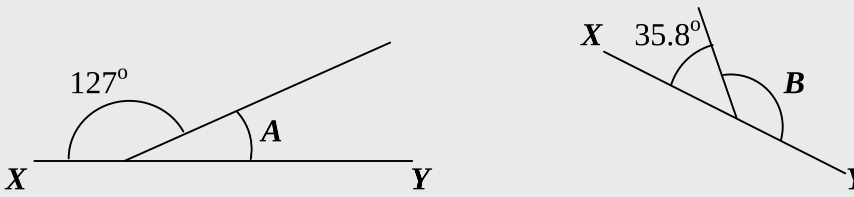


5. Assuming again, a rectangle gets folded the way below, and the angle A is the same as the angle A above, find the angle B.



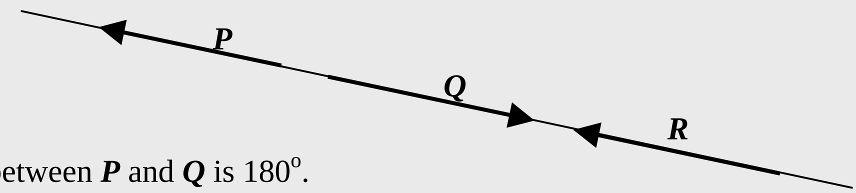
Suggestions or Solutions
To the Problem 0

Assuming *XY* is a line segment, find the two angles *A* and *B*.



By definition, the angle between two line segments that are in the same line is 180^o. So the angle between two rays that are in the same line is 180^o if their directions are opposite. If however, their directions are the same, the angle is 0^o.

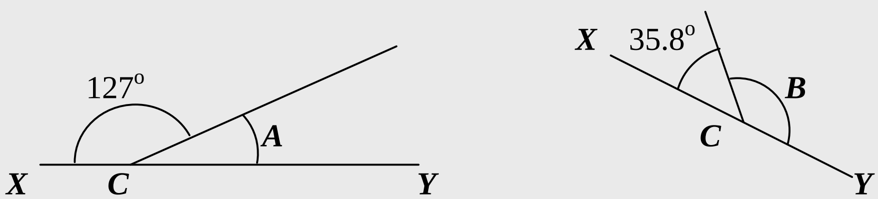
Fig. 0.0



So the angle between ***P*** and ***Q*** is 180^o.
And the angle between ***Q*** and ***R*** is 180^o, too.
But the angle between ***P*** and ***R*** is 0^o.
Assuming however, ***P***, ***Q***, and ***R*** are *line segments*, we say that the angle between any two of the three is 180^o.

Thus, in the figure below, assuming ***C*** is a point in the line segment ***XY***, we can say that the angle between ***CX*** and ***CY*** is 180^o.

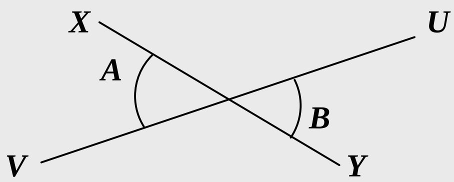
Fig. 0.1



So we get $\mathbf{127^o} + \boldsymbol{A} = \mathbf{180^o} \Rightarrow \boldsymbol{A} = \mathbf{53^o}$, and $\mathbf{35.8^o} + \boldsymbol{B} = \mathbf{180^o} \Rightarrow \mathbf{B} = \mathbf{144.2^o}$.

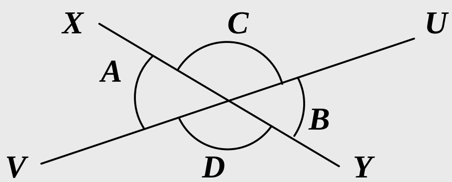
Suggestions or Solutions
To the Problem 1

Assuming *XY* and *UV* are two line segments, show that the two angles *A* and *B* are the same



We know that ***XY*** and ***UV*** both are line segments.
So in the figure below, we get $A + C = 180^o$, and $B + C = 180^o$.

Fig. 1.0



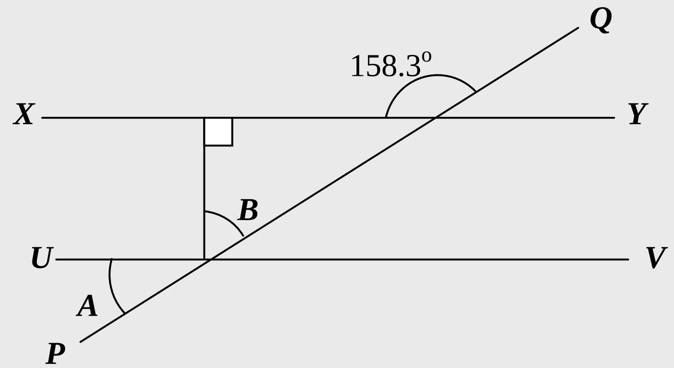
Thus, we get $A + C = B + C \Rightarrow A = B$.
And by the same token, we can say that $C = D$, too.

And in particular, such two angles as ***A*** and ***B*** are called vertical angles, opposite angles, or vertically opposite angles. So vertical angles or opposite angles are the same. Thus for instance, ***C*** and ***D*** are called vertical angles or opposite angles. And the angle ***A*** is said to be the vertical angle of the angle ***B***. In short, ***A*** is vertical of ***B***.

By the way, if two angles add up to 180^o, the two are said to be supplement to each other. So for instance, the two angles ***A*** and ***C*** above are supplement to each other, and the angle ***D*** is supplement to the angle ***A***. And the same is true, too, for the two angles ***B*** and ***C***, and also, for the two angles ***B*** and ***D***.
If however, two angles add up to 90^o, the two angles are said to be complement to each other. So for instance, if $P + Q = 90^o$, ***P*** is complement to ***Q***, and vice versa.

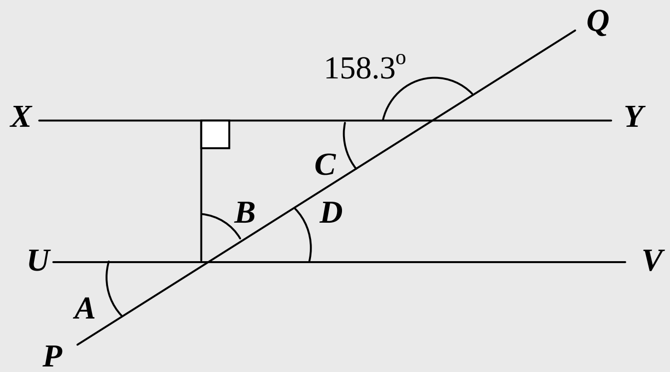
Suggestions or Solutions
To the Problem 2

Assuming *XY* is parallel to *UV*, find the two angles *A* and *B*.



Indicating two angles ***C*** and ***D*** the way below, we can say that the angle ***C*** is supplement to the angle 158.3°, and the angle ***D*** is complement to the angle ***B***, because ***XY*** is parallel to ***UV***.

Fig. 2.0



So we get $C + 158.3^\circ = 180^\circ$, and $B + D = 90^\circ$. Thus, we can get first $C = 180^\circ - 158.3^\circ = 21.7^\circ$. How then can we get the angle ***B***?

We know that the sum of the three angles in a triangle is 180°, and that ***B*** is one of the three angles in a right triangle, where one angle is 90°.

So we get

$$B + C + 90^\circ = 180^\circ \Rightarrow B = 180^\circ - C - 90^\circ = 180^\circ - 21.7^\circ - 90^\circ = 180^\circ - 111.7^\circ = 68.3^\circ.$$

Next, we have ***B*** + ***D*** = **90º**. So we get ***D*** = **90º** – ***B*** = **90º** – **68.3º** = **21.7º**.

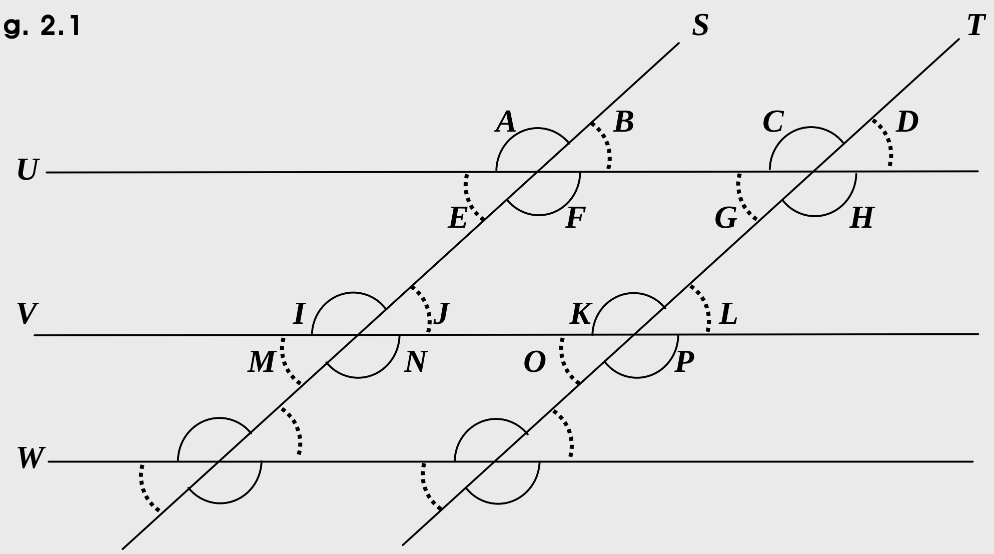
And next, we know that the angle ***A*** is vertical of the angle ***D***. That is, ***A*** = ***D***.
So we get ***A*** = **21.7º**.

We can notice the angle ***A*** is the same as the angle ***C***, which is not coincidental.
Both the two angles are equal, because the two lines ***XY*** and ***UV*** are parallel.
And we call such angles are corresponding angles, which are therefore, equal.

And also, we can notice that the angle ***C*** is the same as the angle ***D***, which is not coincidental either. That's also because the two lines ***XY*** and ***UV*** are parallel.
And we call such angles are alternate angles, which are therefore, equal, also.

So suppose for instance, that in the figure below, the three lines ***U***, ***V***, and ***W*** are parallel to each other, and also, that the two lines ***S*** and ***T*** are parallel.

Fig. 2.1

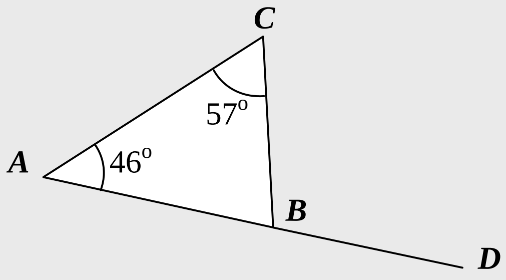


Then, we can say that ***A***, ***I***, ***C***, and ***K*** are corresponding angles, ***E***, ***M***, ***G***, and ***O*** are corresponding angles, ***A*** and ***N*** are alternate angles, and ***E*** and ***J*** are alternate angles.
And we know that the two angles ***A*** and ***F*** are vertical angles, and thus, are equal.

So we get ***A*** = ***C*** = ***F*** = ***H*** = ***I*** = ***K*** = ***N*** = ***P***, and ***B*** = ***D*** = ***E*** = ***G*** = ***J*** = ***L*** = ***M*** = ***O***.

Suggestions or Solutions
To the Problem 3

Assuming the line segment AD includes the side AB in ΔABC, find $\angle CBD$.



We know the angle between AB and BD is 180^o, because AB and BD are in a line.

So knowing $\angle ABC$, we can get $\angle CBD$, since $\angle ABC + \angle CBD = 180^o$.
How then can we get $\angle ABC$?

We know that the sum of the three angles in a triangle is 180^o.
So we get $57^o + 46^o + \angle ABC = 180^o \Rightarrow \angle ABC = 180^o - 103^o = 77^o$.

Thus, getting back to $\angle ABC + \angle CBD = 180^o$, we get

$\angle CBD = 180^o - \angle ABC = 180^o - 77^o = 103^o$.

And we can notice that $\angle CBD$ is the sum of the two angles 57^o and 46^o, that is, the sum of the two angles $\angle CAB$ and $\angle CBA$, which is not coincidental, though.

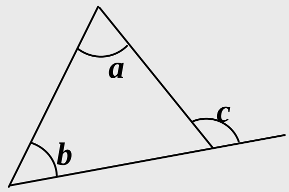
Assuming $X = \angle CAB + \angle BCA$, we get $X + \angle ABC = 180^o = \angle CBD + \angle ABC$.

So we get $X = \angle CBD \Rightarrow \angle CAB + \angle BCA = \angle CBD$.

And we know that $\angle ABC$ and $\angle CBD$ are supplement to each other, because we have $\angle ABC + \angle CBD = 180^o$. So we can say that $\angle CBD$ is supplement to $\angle ABC$.

Thus, we can say that in a triangle, the sum of two internal angles is the same as the external angle supplement to the other internal angle.

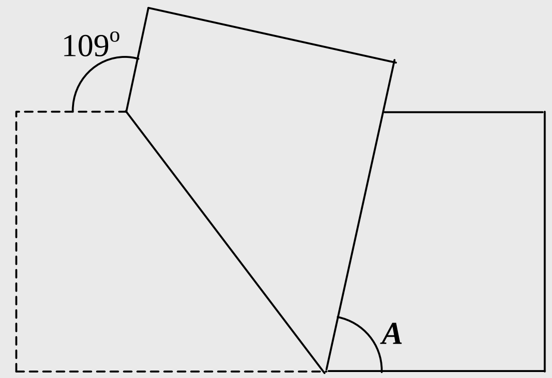
Fig. 3.0



So in the figure above, we get ***a*** + ***b*** = ***c***.

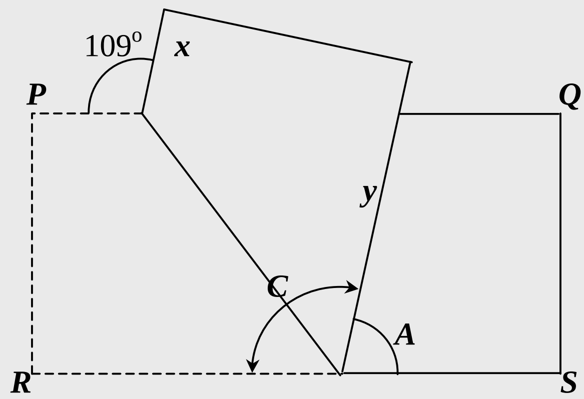
Suggestions or Solutions
To the Problem 4

Assuming a rectangle gets folded the way below, find the angle *A*.



We know that the two sides facing each other in a rectangle are not only the same but parallel, too. So in the figure below, we have $x \,//\, y$, and $PQ \,//\, RS$.

Fig. 4.0

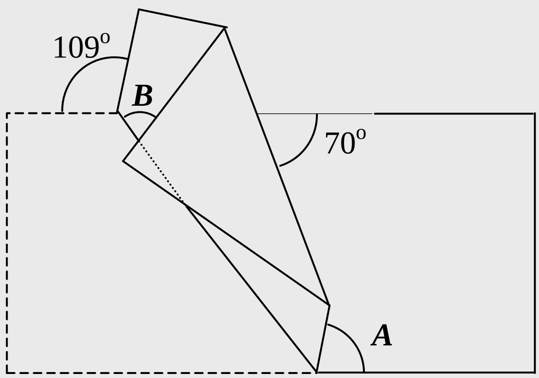


So we can say that the angle 109° and the angle C are corresponding angles, and thus, are equal. That is, we get $C = 109°$.

And we know $C + A = 180°$. So we get $A = 180° - C = 180° - 109° = 71°$.

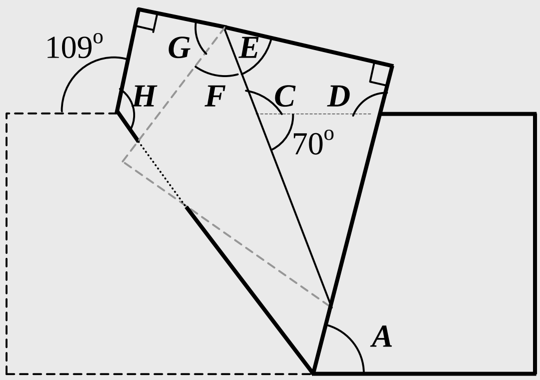
Suggestions or Solutions
To the Problem 5

Assuming again, a rectangle gets folded the way below, and the angle *A* is the same as the angle *A* above, find the angle *B*.



Unfolding the figure above, we can unfold the way to the solution. So unfolding it the way below, we can see some angles that can help find the solution.

Fig. 5.0



Then, to begin with, we get $\boldsymbol{C} + \mathbf{70^o} = \mathbf{180^o} \Rightarrow \boldsymbol{C} = \mathbf{110^o}$.

Next, we get $\boldsymbol{D} = \mathbf{109^o}$, because $\boldsymbol{D}$ and 109^o are corresponding angles, and thus, are equal. And we know that $\boldsymbol{C} + \boldsymbol{D} + \boldsymbol{E} + \mathbf{90^o} = \mathbf{360^o}$ since the sum of all the four angles in a quadrangle is 360^o.

So we get $\boldsymbol{E} = \mathbf{360^o} - \mathbf{90^o} - \boldsymbol{C} - \boldsymbol{D} = \mathbf{270^o} - \mathbf{110^o} - \mathbf{109^o} = \mathbf{51^o}$.

Next, we get $E = F$, because of unfolding. So we get $F = 51^o$, too.

Next, we have $E + F + G = 180^o$. So we get $G = 180^o - E - F = 180^o - 102^o = 78^o$.

Next, we get $B + G + 90^o + H = 360^o$. How?

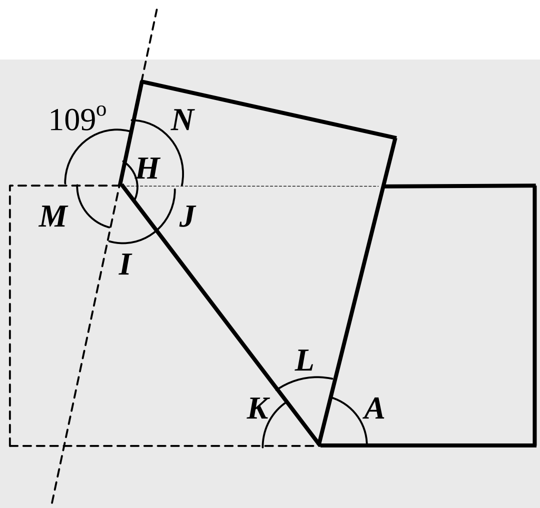
That's because B is one of the four angles in a quadrangle, and the sum of all the four angles in a quadrangle is 360^o. So we get $B + G + 90^o + H = 360^o$.

In other words, we get $B = 360^o - G - 90^o - H = 360^o - 78^o - 90^o - H = 192^o - H$.

So next, finding H, we can get B, which is the solution.
How can we get H though?

Unfolding the figure above, we can unfold the way to the solution.
So next, unfolding it the way below, we can see some angles that can help find the solution.

Fig. 5.1



To begin with, we can see $K = L$, because after unfolding, L is the same as K.

Next, we know K and J are alternate angles, and so are I and L.
So we get $K = J$, and $I = L$. Thus, we get $I = J$, since $K = L$.

And we have $I + J = 109^o$, because $I + J$ and 109^o are vertical angles.

So we get $2J = 109^o$, since $I = J$. Thus, we get $J = 54.5^o$.

Next, we have $M + 109^o = 180^o$. So we get $M = 71^o$.

Next, we know M and N are vertical angles. So we get $M = N = 71^o$.
What then, is the angle H?

It is the sum of N and J, so we get $H = 71^o + 54.5^o = 125.5^o$.

Now, we have $B = 192^o - H$. So we get $B = 192^o - 125.5^o = 66.5^o$.